SIMULADOR DE TIEMPOS POR VUELTA

Laptime simulation:

Laptime simulation is a useful tool to understand the impact that every factor has on laptime in order to decide whether to define a starting point for the setting of the vehicle or on which parameters to focus on or to define strategies.

The aim of this exercise is for the student to understand how simulation can assist in making decisions by analysing the importance of each of the following factors on laptime, the factors are: Vehicle mass; Engine power; CG height; Track width; Downforce and Aero efficiency.
Build a spreadsheet in Excel and vary each variable + and – 10%, record the change in laptime and present the results as clear as possible. State any conclusions from the results.

Circuit:

The circuit is comprised of three straights of 500m each intercalated with three corners all with different radius.

Variables:
Vehicle Mass, m = 650 kg
Engine Power, P = 600 kW
CG Height, h = 0,27 m
Track Width, track = 1,5 m
Downforce, downforce = 8,5 kN
Aero Efficiency, aero_eff = 3,5
Drag Reference, drag_ref = downforce/aero_eff (units in kN)
Reference Velocity, V_ref = 70 m/s
Corner radius, *R*:
Corner 1 = 130 m
Corner 2 = 190 m
Corner 3 = 250 m

Assumption: All the static weight is equally distributed over all the wheels
The downforce is also equally distributed over all the wheels
For the corner distance assume that each corner is 1/3 of the full circle.
Reference velocity is the velocity at which the downforce is quoted.

Because aerodynamic forces change proportionally to the square of the velocity, quoting force at a given velocity (V_ref) allows calculating the aerodynamic force at any velocity.

Aero efficiency is the ratio between downforce and drag

CG Height is the height of the centre of gravity of the vehicle

Cornering:

The cornering speed is either limited by the grip that the tyres can generate (traction limited) or by the power produced by the engine (power limited).

Power Limited:

The engine cannot produce enough power to overcome the drag at the maximum cornering speed allowed by the grip of the tyres.

$$Available\_Power = \frac{P \times 1000 - D_{total} \times V}{1000} \quad [kW] \ldots\ldots\ldots\ldots (1)$$

Where:

$$D_{total} = D_{tyre} + D_{aero} \quad [N] \ldots\ldots\ldots\ldots (2)$$

D_{total}, total drag force
D_{tyre}, drag from the tyres
D_{aero}, aerodynamic drag
V, velocity

$$D_{aero} = \frac{V^2}{V\_ref^2} \times drag\_ref \times 1000 \quad [N] \ldots\ldots\ldots\ldots (3)$$

$$D_{tyre} = m \times \frac{v^2}{R} \times \tan\alpha + C_r \times m \times g \quad [N] \ldots\ldots\ldots\ldots (4)$$

Where Cr is the rolling coefficient (use *Cr* = 0.02) and a is the slip angle at which the tyre generates maximum lateral force (Assume a = 6º).

Traction limited:

$$F_{y\_max} = F_{y\_required} \quad [N] \ldots\ldots\ldots\ldots (5)$$

$F_{y\_required}$ is the force that the tyres have to match for the system to attain equilibrium, in this case the system is the vehicle cornering at the limit of ; this force comes from lateral acceleration.

$$F_{y\_required} = m \times a_y = m \times \frac{V^2}{R} \quad [N] \ldots\ldots\ldots\ldots (6)$$

F_{y_max} is the maximum lateral force that all four tyres can generate

During cornering, the tyres on the side of the car closer to the centre of the corner are called inside tyres, and the tyres on the side of the car farther to the centre of the corner are called outside tyres

$$F_{y\_max} = F_{yo} \times 2 + F_{yi} \times 2 \quad [N] \text{ .... (7)}$$

F_{yo} is the lateral force generated by each of the outside tyres.
F_{yi} is the lateral force generated by each of the inside tyres.

The maximum lateral force generated from a tyre can be derived from the vertical force at the *contact patch**.

* the contact patch is the contact surface between the tyre and the ground

$$F_y = F_z \times \mu \quad [N] \text{ .... (8)}$$

$$\mu = 1.82 - \frac{\left(\frac{F_z}{1000}\right)^2}{45} \quad \text{...................... (9)}$$

The equation to obtain m for this case is an empirical equation obtained experimentally. Although this equation is only valid for a specific tyre it implies that m varies with load which is applicable for all tyres.

The vertical force acting on the ground at the contact patch is the sum of aerodynamic, static and dynamic forces (load transfer)

$$F_z = F_{z\_aero} + F_{z\_static} + F_{z\_load\_transfer} \quad [N] \text{ .... (10)}$$

The difference between the vertical loads on the inside and outside tyres is the load transfer during cornering. For F_{zo} load transfer is positive and for F_{zi} load transfer is negative.

$$F_{Z\_Load\_transfer} = \frac{m \times h \times V^2}{2 \times track \times R} \quad [N] \text{ .... (11)}$$

Knowing that the static weight is equally distributed over the 4 wheels, static load at each wheel is:

$$F_{z\_static} = \frac{m \times g}{4} \quad [N] \text{ .... (12)}$$

Similarly to Aerodynamic drag (equation (3)), the Aerodynamic load is:

$$F_{z\_aero} = \frac{\frac{V^2}{V\_ref^2} \times downforce \times 1000}{4} \quad [N] \text{ .... (13)}$$

With the difference that the reference force used is the *downforce* instead of *drag_ref* and is divided by 4 to get the load per wheel (remember that the downforce is also equally distributed by all the wheels).

Straight Line:

Acceleration:
For simplicity assume that the acceleration is only limited by engine power and not by traction. Longitudinal acceleration, a_x is:

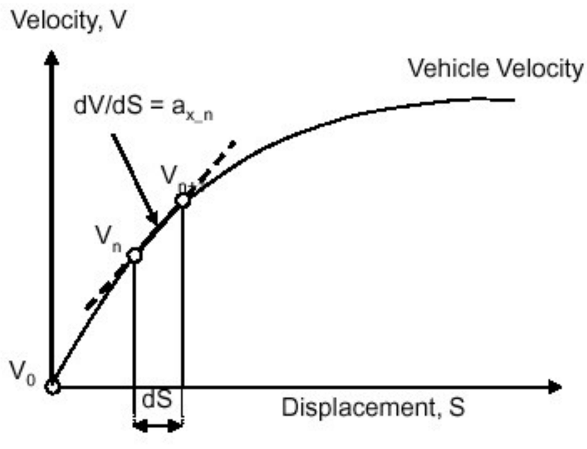
$$a_x = \frac{\frac{P}{V} - Drag}{m}$$ [m/s²] .... (14)

Where *Drag* is the vehicle drag in straight line.
Note: remember that engine power must be in watts.

For simplicity, assume that there is no drag from tyre slip. Thus the drag in straight line is the same as D_{total} in equation (2) without the term with slip angle, a (equation (4)).

$$Drag = \frac{V^2}{V\_ref^2} \times drag\_ref \times 1000 + C_r \times m \times g$$ [N] .... (15)

As the rate of acceleration changes continuously, we can approximate the result by dividing the velocity in small increments of displacement at which the acceleration (a_x) is assumed constant. See the graph below (*Graph 1*).



Graph 1- Velocity vs. displacement in straight line acceleration.

V_0 is the cornering speed.

The velocity is calculated starting from cornering speed where:

$V_{n+1} = V_n + a_{x\_n} \times dt$ [m/s] .... (16)

Where *dt* is the time split and can be calculated as follows:

$dt = dS / Vn$ [s] .... (17)

This assumes that for a given time interval, the vehicle is travelling that distance at constant speed which is not true but if we divide the displacement in small increments (e.g. 5m for straights that are 500m long) we get only a small error associated with it and for this exercise this is an approximation good enough.

Note: The important part is that we are aware of all the assumptions we make along the way and understand the risks associated with it.

Braking:

In this case we assume that braking is predominantly determined by drag and tyre grip and that longitudinal tyre grip is proportional to vertical load. Also static loads and load transfer effects on tyre vertical load were ignored for simplicity.

$$braking = \frac{\left[\left(\frac{V}{V\_ref}\right)^2 \times downforce \times 1000 + m.g\right] \times \mu_{tyre} + Drag}{m}$$ [m/s²].......... (18)

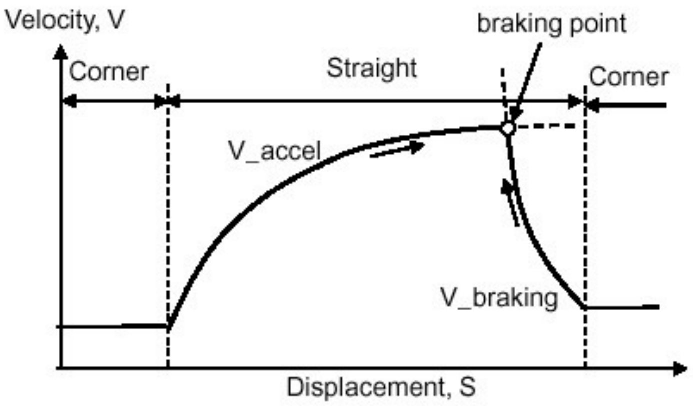
m_{tyre} is the friction coefficient for the tyre when braking (use m_{tyre} = 1.4).

Drag from equation (15)

Also Velocity can be calculated by: $V_{n-1} = V_n + a_{x\_n}.dt_n$ [m/s]..............(19)

And time split, $dt_n = S_n - S_{n-1}$ [sec]..............(20)

The braking velocity can be calculated in a similar way as for acceleration. In Excel a method to determine the braking point is to start from the following corner velocity and work backwards.
The displacement point where both acceleration and braking velocities meet is the braking point (see graph 2).



Graph 2- velocity vs. displacement